School Choice: recent development

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Last lecture

- Three mechanisms for school choice
  - deferred acceptance
  - top trading cycles
  - Boston
- Inefficiency in DA
  - priority design
  - school choice with consent

School choice: real-life application of matching

More and more cities around the world use school choice programs:

- school authorities take into account preferences of children and their parents.
- typical goals of school authorities are:
  - (1) efficient placement,
  - (2) fairness of outcomes,
  - (3) easy for participants to understand and use, etc.

Question: could we achieve all these goals? trade-offs?

Characteristics

- indivisibilities,
- one-sided preferences (students),
- no monetary compensation,
- public ownership assorted with priorities

Outline

- Sincere and sophisticated students
- Indifferences in priorities
- Manipulability

Background

A school district asks parents or students for their preferences

- each school has limited seats,
- all students cannot get their first choice schools for over-demanded schools.
- the district has to reject some students
- efficient, fair and lawsuit-free mechanisms are not trivial,
- design is required

Model

A school choice problem is a triple $(I, S, P, \succ, q)$ where:

- I is a set of students
- S is a set of schools
- P is a list of preferences over $S \cup \{\emptyset\}$
- $\succ$ is list of priorities over I
- q is a vector of positive numbers

Assumption

- *We assume that preferences and priorities are strict,*
- $|I| \leq \sum_{s \in S} q_s$.

Model

- A matching is a function $\mu : I \to S \cup \{\emptyset\}$ such that for each school s, $|\mu^{-1}(s)| \leq q_s$.
- A mechanism assigns each pair $(P, \succ)$ a matching.

Design goals

- Individual rationality

Definition

A matching μ is individually rational if for each student i

$$\mu(i)\ R_i\ \emptyset$$

- Elimination of justified envy

Definition

A matching μ eliminates justified envy if for each $i \in I$, there is no $j \in I$

$$s\ P_i\ \mu(i), \quad \mu(j) = s \quad \text{and} \qquad i \succ_s j.$$

Such a matching is said to be fair.

Design goals

- Non-wastefulness

Definition

A matching μ is non-wasteful if for each student i and each school s

$$s \; P_i \; \mu(i) \quad \Rightarrow \quad |\mu^{-1}(s)| = q_s.$$

- Stability

Definition

A matching is stable if it is individually rational, eliminates justified envy and is non-wasteful.

Design goals

- Strategy-proofness

Definition

A mechanism φ is strategy-proof if for each P and each student i, there is no P_i' such that

$$\varphi_i(P_i', P_{-i}, \succ) \; P_i \; \varphi_i(P, \succ).$$

Student-proposing deferred acceptance

Step 1:

- each student applies to his most preferred acceptable school.
- each school follows its priority and tentatively accepts one at a time its best applicants up to its capacity and rejects the rest.

Step $k, k > 1$

- each student who is rejected at Step $k-1$ applies to his next acceptable school.
- each school considers the new applicants together with those who are tentatively accepted in the previous step, and follows its priority and accepts one at a time, its best applicants up to its capacity and rejects the rest.

The algorithm terminates when every student is tentatively accepted or has applied to all his acceptable schools.

Top trading cycle (TTC) mechanism

Step 1

- Each student points to his most preferred acceptable school. Each school points to the student with the highest priority,
- each student in each cycle is assigned to the school he is pointing to and removed, while the capacity of each of these schools is reduced by one.

Step k, $k > 1$:

- Each remaining student points to his next most preferred acceptable school. Each school with remaining seats points to the student with the highest priority (there is a cycle!)
- each student in each cycle is assigned to the school he is pointing to and removed, while the capacity of each of these schools is reduced by one.

Boston mechanism: a mechanism from practice

Step 1:

- Each student applies to his first acceptable choice school,
- each school follows its priority and immediately accepts one at a time its best applicants until up to its capacity and rejects the remaining applicants

Step $k, k > 1$*:*

- each student who is rejected at Step $k-1$ applies to his k'th acceptable choice,
- each college follows its priority and immediately accepts its best new applicants up to its remaining seats.

The algorithm terminates when each student has been accepted or has been rejected by all his acceptable schools or no school has remaining seats.

West Zone Parents Group in Boston

It is a well-informed group of approximately 180 members who meet regularly prior to admissions time to discuss Boston school choice for elementary school.
Their introductory meeting minutes on October 27, 2003, state:

"One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a safe second choice."

Evidence from data: there are different levels of sophistication among the families who participate in the mechanism.

BM: sincere and sophisticated students

The West Zone Parents Group in Boston opposed to changes in 2005: "Dont change the algorithm, but give us more resources so that parents can make an informed choice" (public hearing, June 8, 2005).

Goal:

- identify the Nash equilibria of the Boston game
- compare the outcome for each sincere student to the outcome when he becomes sophisticated,
- compare the equilibrium outcomes for sophisticated students to the outcome of the dominant-strategy outcome of DA.

Nash equilibria

Sincere who ranked s first + sophisticated

Sincere who ranked s second

Sincere who ranked s third

⋮

Sincere who ranked s last

Nash equilibria

Let $\succ^{aug}$ be an augmented priority constructed as follows:

- Each student in a given block has higher priority than each student in a lower block,
- in each block, students are ordered according to $\succ_s$.

Nash equilibria

Theorem (Pathak & Sonmez, 2008)

The set of Nash equilibrium outcomes of the Boston game under $(P, \succ)$ is equivalent to the set of stable matchings of $(P, \succ^{aug})$.

There is a Pareto-optimal Nash equilibrium outcome: the student-optimal stable matching of $(P, \succ^{aug})$.

Becoming sophisticated

Theorem (Pathak & Sonmez, 2008)

Every sincere student receives the same outcome at every Nash equilibrium outcome of the Boston game.

Proof.

□

Theorem

Every sincere student weakly benefits from becoming sophisticated in the Pareto-dominant Nash equilibrium of the Boston game, whereas all other sophisticated students weakly suffer.

Sophisticated students take advantage over sincere students

Theorem (Pathak & Sonmez, 2008)

The school a sophisticated student receives in the Pareto-dominant equilibrium of the Boston mechanism is weakly better than her dominant-strategy outcome under the student-optimal stable mechanism.

CC: a strategy-proof mechanism levels the playing field.

Proof.

□

Indifferences in priorities

In this section, we assume that each school has a weak priority, that is, there might be ties among some students.

- This is typical in school choice
- How the ties are broken has a welfare implication.

What is the matter of tie-breaking?

Example

P_1	P_2	P_3
s_2	s_2	s_1
s_1	s_1	s_2

$\succ_{s_1}$	$\succ_{s_2}$
$1, 2$	3
3	$1, 2$

Example (Welfare loss with tie-breaking)

P_1	P_2	P_3
s_2	s_2	s_1
s_1	s_1	s_2

$\succ_{s_1}$	$\succ_{s_2}$
$1, 2, 3$	$1, 2, 3$

Definition

A stable matching is student-optimal if it is not Pareto-dominated by another stable matching.

Stable improvement cycle algorithm

For each school s, let D_s denote the set of highest $\succeq_s$-priority students among those who desire s.

A stable improvement cycle consists of distinct students $i_1, \ldots, i_n \equiv i_1$ $(n \geq 2)$ such that

- $\mu(i_\ell) \in S$,
- i_ℓ desires $i_{\ell+1}$ and
- $i_\ell \in D_{\mu(i_{\ell+1})}$.

Given a stable improvement cycle define a new matching μ' by:

$$\mu'(j) = \begin{cases} \mu(i_{\ell+1}) & \text{if } j = i_\ell \\ \mu(j) & \text{if } j \notin \{i_1, \ldots, i_n\}. \end{cases}$$

Stable improvement cycle algorithm

Theorem (Erdil & Ergin, 2008)

If a stable matching μ is Pareto dominated by another stable matching, then it admits a stable improvement cycle.

Stable improvement cycle algorithm

Step 0: Select a strict priority structure. Run the DA algorithm and obtain a temporary matching μ^0.

Step $t-1$*:*

(t:a) Given μ^{t-1}, let the schools stand for the vertices of a directed graph, where for each pair of schools s_1 and s_2, there is an edge $s_1 \rightarrow s_2$ if and only if there is a student i who is matched to s_1 under μ^{t-1}, and $i \in D_{s_2}$.

(t.b) If there are any cycles in this directed graph, select one. For each edge $s_1 \rightarrow s_2$ on this cycle select a student $i \in D_{s_2}$ with $\mu^{t-1}(i) = s_1$. Carry out this stable improvement cycle to obtain μ^t, and go to step (t+1:a). If there is no such cycle, then return μ^{t-1} as the outcome of the algorithm.

Manipulability in practice

There has been reforms in school choice due to excessive manipulation:

- In June 2005, the BPS voted to replace their mechanism with a version of DA,
- in 2009, Chicago Public Schools changed their mechanisms halfway through running it,
- in 2010, local authorities in England abandon their mechanism which is a hybrid between Boston and DA.

Unfortunately, the new mechanisms was also manipulable. However, they were perceived to be less manipulable than the oldest ones.

Comparing mechanisms by their vulnerability to manipulate

For simplicity we assume that each school has a strict priority.
A mechanism φ is manipulable at P if there is a student i and P'_i such that

$$\varphi_i(P'_i, P_{-i})\ P_i\ \varphi_i(P).$$

Given a mechanism φ, let M^{φ} denote the set of profiles where φ is manipulable.

Definition (Pathak & Sonmez, 2013)

- A mechanism ϕ is at least as manipulable as φ if $M^{\phi} \supseteq M^{\varphi}$.
- A mechanism φ is less manipulable than ϕ if $M^{\varphi} \subsetneq M^{\phi}$.

Is this notion relevant? Are there other compelling notions of manipulability?

Constraint school choice

- In practice, students are required to rank a limited number of schools. This practice introduces manipulability in DA and Boston.
- For each mechanism φ, let φ^k denote the mechanism where each student is required to submit at most k acceptable schools.

Constraint school choice

First preference first mechanism:
some schools are equal preference schools and priorities need to be respected and the other schools are first preference schools, where the ranking overrides priorities (much like Boston).

Let *FPF* denote this mechanism.

Comparing mechanisms

Theorem (Pathak & Sonmez, 2013)

- *Let $\ell > k > 0$ and suppose there are at least ℓ schools. Then DA^{ℓ} is less manipulable than DA^{k}.*
- *Suppose there are at least k schools where $k > 1$. Then DA^{k} is less manipulable than FPF^{k}.*
- *Suppose there are at least k schools where $k > 1$. Then DA^{k} is less manipulable than BM^{k}.*

Table 1—School Admissions Reforms

Allocation system	Year	From	To	Manipulable (More or less?)	Source
Boston Public Schools (K, 6, 9)	2005	Boston	GS	Less	A,B,E
Chicago Selective High Schools	2009	Boston[4]	SD[4]	Less	A,B,C
	2010	SD[4]	SD[6]	Less	A,B,C
Ghana—Secondary schools	2007	GS[3]	GS[4]	Less	E
	2008	GS[4]	GS[6]	Less	E
Denver Public Schools	2012	Boston[2]	GS[5]	Less	A,B
Seattle Public Schools	1999	Boston	GS	Less	A,B,C,E,F
	2009	GS	Boston	More	A,B,C,F
England					
Bath and North East Somerset	2007*	FPF[3]	GS[3]	Less	A,D
Bedford and Bedfordshire	2007*	FPF[3]	GS[3]	Less	A,D
Blackburn with Darwen	2007*	FPF[3]	GS[3]	Less	A,D
Blackpool	2007*	FPF[3]	GS[3]	Less	D
Bolton	2007*	FPF[3]	GS[3]	Less	A,D
Bradford	2007*	FPF[3]	GS[3]	Less	A,D
Brighton and Hove	2007	Boston[3]	GS[3]	Less	A,C,D,E
Calderdale	2006	FPF[3]	GS[3]	Less	A,C
Cornwall	2007*	FPF[3]	GS[3]	Less	D
Cumbria	2007*	FPF[3]	GS[3]	Less	D
Darlington	2007*	FPF[3]	GS[3]	Less	D
Derby	2005*	FPF[4]	GS[4]	Less	A,D
Devon	2006*	FPF[3]	GS[3]	Less	A,D
Durham	2007	FPF[3]	GS[3]	Less	A,D
Ealing	2006*	FPF[6]	GS[6]	Less	A,D
East Sussex	2007	Boston[3]	GS[3]	Less	A,D
Gateshead	2007*	FPF[3]	GS[3]	Less	D
Halton	2007*	FPF[3]	GS[3]	Less	A,D
Hampshire	2007	FPF[3]	GS[3]	Less	A,D
Hartlepool	2007	FPF[3]	GS[3]	Less	A,D
Isle of Wright	2007*	FPF[3]	GS[3]	Less	D
Kent	2007	Boston[3]	GS[4]	Less	A,D
Kingston upon Thames	2007*	FPF[3]	GS[4]	Less	A
Knowsley	2007*	FPF[3]	GS[3]	Less	A,D
Lancashire	2007*	FPF[3]	GS[3]	Less	A,D
Lincolnshire	2007*	FPF[3]	GS[3]	Less	A,D
Luton	2007*	FPF[3]	GS[3]	Less	D
Manchester	2007*	FPF[3]	GS[3]	Less	A,D
Merton	2006	FPF[6]	GS[6]	Less	A,D
Newcastle	2005	Boston[3]	GS[3]	Less	A
	2010	GS[3]	GS[4]	Less	A

Taiwan mechanism

- Students have scores and there are deductions schemes. Let λ be a $m+1$-vector such that $\lambda_1 = 0$ and $\lambda_t \leq \lambda_{t+1}$ for each $t < m+1$.
- If a student ranks school s at ℓ'th position, then his score at school s is deducted by λ_ℓ.
- After adjusting scores, run DA with the induced priorities.

We write $\gamma > \lambda$ if for each t, $\gamma_t \geq \lambda_t$ and for some t, $\gamma_t > \lambda_t$.

Theorem (Dur et al., 2018)

Suppose that students have the same scores. Then, if $\gamma > \lambda$, then the Taiwan mechanism with deduction rule λ is less manipulable than Taiwan mechanism with deduction rule λ.

Other models

- multiple versus single tie-breaking
- Chinese mechanism
- School choice with affirmative action
- decentralized matching
- random path to stability
- strategic schools, manipulation via capacities and pre-arrangement
- etc.

Next lecture: kidney exchange!