Kidney exchange

Somouaoga BONKOUNGOU

National Research University Higher School of Economics

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Last lecture: school choice

- Sincere and sophisticated students under the Boston mechanism
  - sophisticated students take advantage over sincere students
  - every sincere student becomes weakly better off when he becomes sophisticated
- Ties in priorities
  - DA under fixed tie-breaking does not yield student-optimal stable matchings
  - stable improvement cycle algorithm
- Manipulability
  - comparing mechanisms by comparing profiles where they are manipulable
  - observation from the field: transition to less manipulable mechanisms

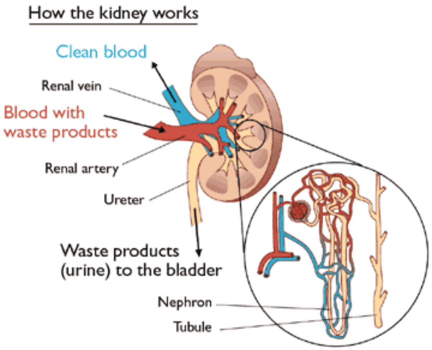
Outline

- Introduction
- Model
- The exchange mechanism
- Properties

Function of a kidney

A kidney cleans the blood:

- blood with waste enters
- clean blood is released
- waste is released through urine



Kidney failure

There are serious kidney diseases: solutions

Dialysis

Transplantation

Some history on transplantation

- The first transplantation from a deceased human donor: soviet surgeon Yurii Voronoy (1939),
- the first paper outlining the concept of paired exchange: Rappaport published in 1986,
- the first recorded paired exchange transplants: organized in South Korea by Dr. Park in 1991

Transplantation is preferred

- Dialysis is costly: in the U.S, $80,000 or more per year per patient.
- Transplantation is preferred: immunosuppressants are affordable

Kidney for transplantation is scarce

More than 100,000 people waiting for a kidney transplant in the U.S (as of January 2016).

Each year,

- nearly 4,000 new patients need a transplantation;
- almost 5,000 patients die waiting for a kidney.

Two sources of kidney supply

- From a deceased donor.

  Managed using a waiting list that takes into account:
  - Waiting time
  - State of the patient
  - other factors (e.g., location, availability of the patient, etc).

- From a living donor.

  The human body has two kidneys but most of us can live with only one.

Transplantation in the US

In 2014, 17,107 kidney transplants took place in the US:

- 11,570 transplants from deceased donors;
- 5,537 transplants from living donors.

The design problem

- Donations from the deceased are not targeted: once we have a kidney we just have to identify a compatible patient.
- Living donors are intended to recipients only.
- Problem: the donor's kidney may not be compatible with his intended recipient.

Kidney compatibility

Two factors determine whether a kidney is compatible for a patient:

- Tissue type compatibility

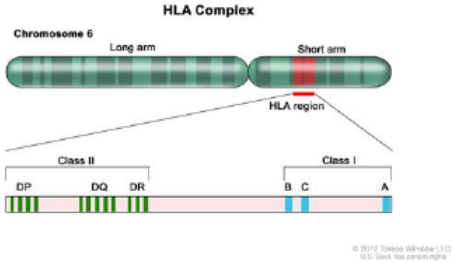
  Relates to the immunological system (set of markers on cells used to detect what is foreign).

- Blood type compatibility

  The blood comes in different types, depending on some proteins we have (or don't have) on our red cells.

Tissue types

Tissue type or Human Leukocyte Antigen (HLA) type: Combination of several pairs of antigens on Chromosome 6. HLA proteins A, B, and DR are especially important.



The immune system uses the HLAs to differentiate self cells and non-self cells. Any cell matching that person's HLA type is not an intruder.

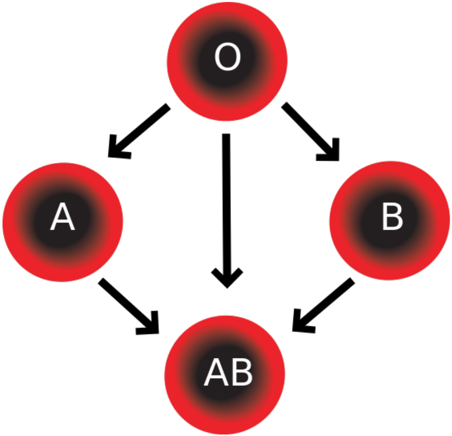
Cross matching test

Prior to transplantation, the potential recipient is tested for the presence of preformed antibodies against donor HLA. If there is a positive crossmatch, the transplantation cannot be carried out.

Blood types

There are 4 major blood types: A, B, AB and O. They correspond to some proteins ("markers") on our red cells:

Blood compatibility



Institutional constraint

According to the 1984 National Organ Transplant Act (NOTA), paying for an organ for transplantation is a felony:

"It shall be unlawful for any person to knowingly acquire, receive or otherwise transfer any human organ for valuable consideration for use in human transplantation."

Except the Republic of Iran, paying for a kidney is not allowed.

Indirect exchange

A first solution is to implement indirect exchanges

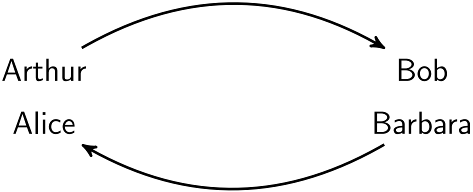
If a the kidney of a patient's donor is not compatible with the patient, then:

- Treat the kidney as a deceased kidney: give it to the highest priority compatible patient in a waiting list;
- Give the patient a high priority position on the waiting list.

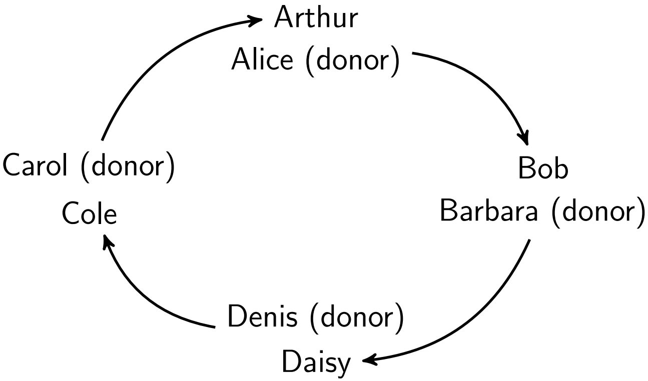
Trading kidneys

The basic idea of trading kidneys is very simple.

- Alice (a patient) and Arthur (her donor), but his kidney is not compatible for her;
- Bob (a patient) and Barbara (his donor), but her kidney is not compatible for him;
- However, Barbara kidney's is compatible for Alice, Arthur's kidney is compatible for Bob.



There's no need to restrict to two couples. . .



With such trades no monetary payment is needed.

The difficulty lies in combining in the same mechanism

- trades;
- the waiting list.

It looks like a problem of assigning objects (i.e., kidneys) with mixed endowment where we would have:

- **Private endowments**: donors' kidneys are "privately held" by the patients;
- **Public endowments**: kidneys from deceased donors.

But it is different from a (classic) problem with mixed endowment: kidneys from deceased donors cannot be considered as available (but not yet assigned).

→ kidneys from deceased donors are not part of the public endowment.

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The design

So sum up, we need to design an exchange procedure that combines at the same time:

- Trades between donors and patients;
- The management of the waiting list for cadaveric donors.

Model

A kidney exchange has the following elements:

- N: set of patients (with kidney failure)
- (k_i, t_i): a patient-donor pair
- K_i: living donors compatible with patient i.
- w: priority in the waiting list in exchange for a live kidney.
- P_i: strict preferences over $K_i \cup \{w, k_i\}$

Model: outcome

A matching is an assignment of kidneys/waitlist option to patients such that:

- each patient is either assigned a compatible kidney, or her donors kidney, or the waitlist option, and
- no kidney can be assigned to more than one patient although the waitlist option w can be assigned to several patients.

House allocation with existing tenants

Consider a modified model of the house allocation (or housing market) where some houses H_E are owned (by existing tenants) and others H_V are commonly owned.

- Allocation of university housing where there are occupants and vacated houses.

A generalization of serial dictatorship and top trading cycles:

- there is an order $f(1), \ldots, f(n)$ of agents:
- the agent ordered first $f(1)$ picks his most preferred house, the agent $f(2)$ ordered second picks his mosts preferred object among those remaining until someone asks an occupied house.

You request my house and I get your turn mechanism

- modify the priority by moving the agent whose house is requested to right above the top priority agent

Example

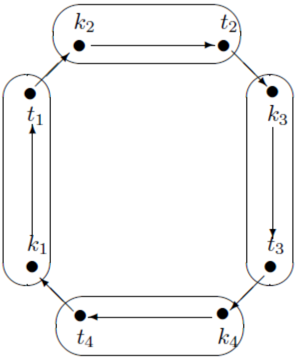
There are five agents $\{1, 2, 3, 4, 5\}$ and five houses $H_E = \{h_3, h_4\}$ owned by agents 3 and 4 respectively; and $H_V = \{a, b, c\}$ vacant houses.

P_1	P_2	P_3	P_4	P_5
a	h_4	a	h_3	b
b	h_3	b	h_4	c
c	c	h_3	a	h_3
h_3	a	c	c	a
h_4	b	h_4	b	h_4

Top trading cycles and chains mechanism

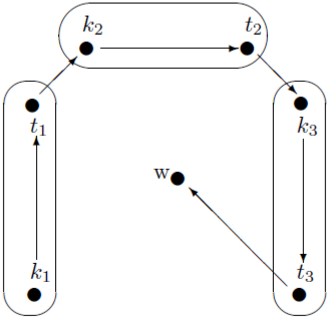
- A *mechanism*: a systematic procedure to select a matching for each kidney exchange problem.
- The Top Trading Cycles and Chains (TTCC) mechanism is based on the following algorithm:
  - each patient t_i points to his or her most preferred option (either towards a kidney in $K_i \cup \{k_i\}$ or towards w), and
  - each kidney k_i points to its paired recipient t_i

Cycles

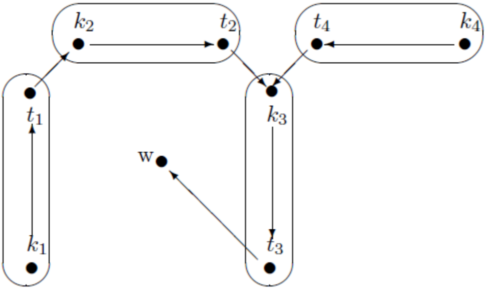


- No two cycles intersects

w-chain



w-chains can intersect



A kidney-patient pair can be part of several w-chains

w-chain selection rule

- We need a rule to select *w*-chains at each step,
- There is a cycle or a *w*-chain at each step.

Important lemma

Lemma (Roth, Sonmez and Unver, 2004)

Consider a graph in which both the patient and the kidney of each pair are distinct nodes, as is the waitlist option w. Suppose each patient points either towards a kidney or w, and each kidney points to its paired recipient. Then

- *either there exists a cycle or,*
- *each pair initiates a w-chain*

The Exchange Mechanism

Step 1: Initially, all kidneys are available, and all agents are active. At each stage:

- each remaining active patient t_i points to his most preferred remaining unassigned kidney or to the wait-list option w, whichever is more preferred
- each remaining passive patient continues to point to his assignment, and each remaining kidney k_i points to its paired recipient t_i.

The Exchange mechanism

Step 2. By the lemma, there is either a cycle, or a w-chain, or both.

- Proceed to Step 3 if there are no cycles. Otherwise locate each cycle and carry out the corresponding exchange. Remove all patients in a cycle together with their assignments
- Each remaining patient points to its top choice among remaining choices and each kidney points to its paired recipient. Proceed to Step 3 if there are no cycles. Otherwise locate all cycles, carry out the corresponding exchanges, and remove them.

Repeat this step until no cycle exists.

The Exchange mechanism

Step 3.

If there are no pairs left, then we are done. Otherwise by the Lemma, each remaining pair initiates a w-chain.

- Select only one of the chains with the chain selection rule.
- The assignment is final for the patients in the selected w-chain.
- In addition to selecting a w-chain, the chain selection rule also determines
  - whether the selected w-chain is removed, or
  - the selected w-chain remains in the procedure although each patient in it is passive henceforth

The Exchange mechanism

Step 4:

Each time a w-chain is selected, a new series of cycles may form. Repeat Steps 2 and 3 with the remaining active patients and unassigned kidneys until no patient is left.

The Exchange mechanism: example

Example (Consider a kidney exchange problem with 12 pairs)

t_1: k_9 k_{10} k_1

t_2: k_{11} k_3 k_5 k_6 k_2

t_3: k_2 k_4 k_5 k_6 k_7 k_8 w

t_4: k_5 k_9 k_1 k_8 k_{10} k_3 w

t_5: k_3 k_7 k_{11} k_4 k_5

t_6: k_3 k_5 k_8 k_6

t_7: k_6 k_1 k_3 k_9 k_{10} k_1 w

t_8: k_6 k_4 k_{11} k_2 k_3 k_8

t_9: k_3 k_{11} ω

t_{10}: k_{11} k_1 k_4 k_5 k_6 k_7 w

t_{11}: k_3 k_6 k_5 k_{11}

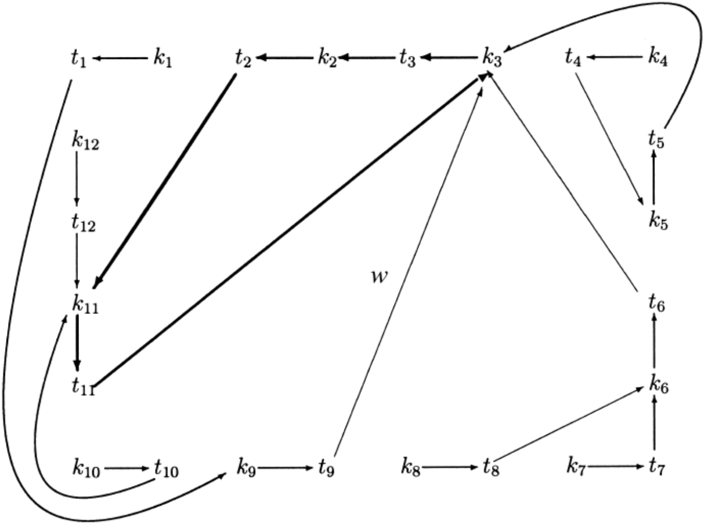
t_{12}: k_{11} k_3 k_9 k_8 k_{10} k_{12} .

Chain selection rule: lower indexed patients are prioritized first

- choose the longest w-chain and keep it until the termination,
- in case the longest w-chain is not unique, choose the w-chain with the highest priority patient;
- if the highest priority patient is part of more than one, choose the w-chain with the second highest priority patient, and so on.

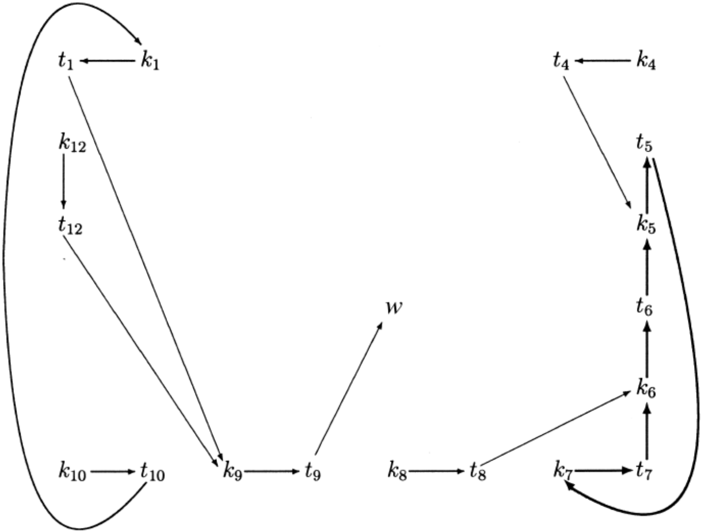
The Exchange mechanism: example

Round 1:



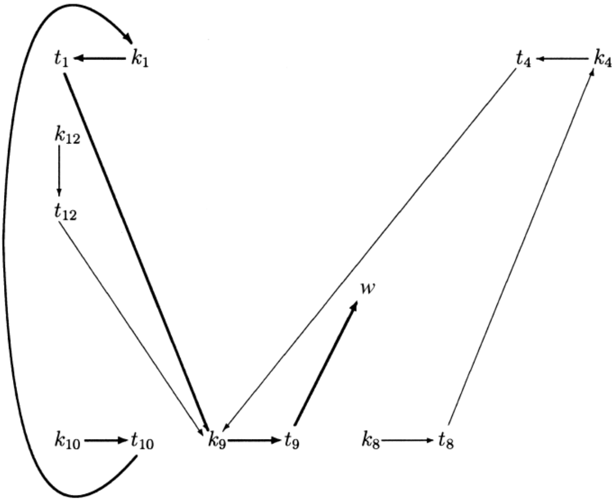
The Exchange mechanism: example

Round 2:



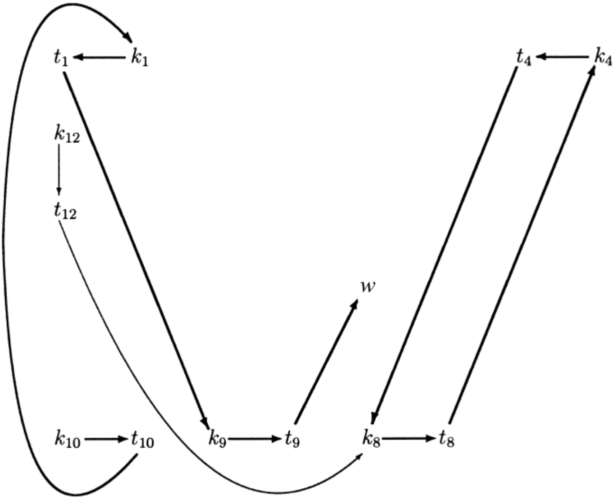
The Exchange mechanism: example

Round 3:



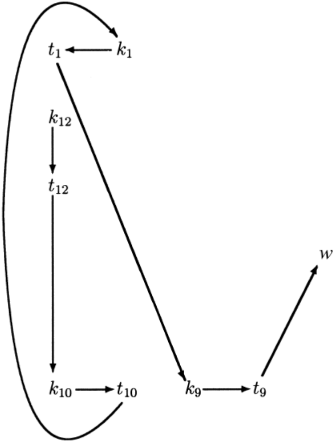
The Exchange mechanism: example

Round 4:



The Exchange mechanism: example

Round 5:



The Exchange mechanism: example

$$\begin{pmatrix} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 & t_9 & t_{10} & t_{11} & t_{12} \\ k_9 & k_{11} & k_2 & k_8 & k_7 & k_5 & k_6 & k_4 & w & \mathrm{k}_1 & \mathrm{k}_3 & \mathrm{k}_{10} \end{pmatrix}$$

Roth, Sönmez and Ünver propose and analyze different selection rules:

Rule a Choose minimal w-chains, and remove them.

Rule b Choose the longest w-chain, and remove it. If the longest w-chain is not unique, then use a tiebreaker to choose among them.

Rule c Choose the longest w-chain, and keep it. If the longest w-chain is not unique, then use a tiebreaker to choose among them.

Rule d Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair, and remove it.

Rule e Prioritize patient-donor pairs in a single list. Choose the w-chain starting with the highest priority pair, and keep it

Definition

A kidney exchange mechanism is efficient if, for any problem (set of patients, donors and their preferences) it always selects an assignment such that:
There does not exist another assignment that is weakly preferred by all patients (and strictly preferred by at least one patient).

Theorem

Consider a chain selection rule such that any chain selected at a nonterminal step remains in the procedure (and thus the kidney of the first donor in the chain remains available for the following steps).
Then the Top Trading Cycle and Chains mechanism, implemented with any such chain selection rule, is efficient.

Incentives

Do patients (e.g., their physicians) have an incentive to report truthfully their preferences?

Theorem

An assignment mechanism that uses the Top Trading Cycles and Chains algorithm with the chain selection rules a, d, or e is strategyproof.

Thank you.